OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

Registered Charity Number: 1066969
[Turn over

1 **(i)** Express $(1 + 8i)(2 - i)$ in the form $x + iy$, showing clearly how you obtain your answer. [2]

(ii) Hence express $\dfrac{1 + 8i}{2 + i}$ in the form $x + iy$. [3]

2 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n + 1)$. [5]

3 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$.

(i) Find the value of the determinant of **M**. [3]

(ii) State, giving a brief reason, whether **M** is singular or non-singular. [1]

4 Use the substitution $x = u + 2$ to find the exact value of the real root of the equation

$$x^3 - 6x^2 + 12x - 13 = 0.$$ [5]

5 Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (8r^3 - 6r^2 + 2r) = 2n^3(n + 1).$$ [6]

6 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.

(i) Find $\mathbf{C}^{-1}$. [2]

(ii) Given that $\mathbf{C} = \mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find $\mathbf{B}^{-1}$. [5]

7 **(a)** The complex number $3 + 2i$ is denoted by w and the complex conjugate of w is denoted by w^*. Find

(i) the modulus of w, [1]

(ii) the argument of w^*, giving your answer in radians, correct to 2 decimal places. [3]

(b) Find the complex number u given that $u + 2u^* = 3 + 2i$. [4]

(c) Sketch, on an Argand diagram, the locus given by $|z + 1| = |z|$. [2]

8 The matrix $\mathbf{T}$ is given by $\mathbf{T} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{T}$. [3]

(ii) The transformation represented by matrix $\mathbf{T}$ is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B, and state the matrices that represent them. [6]

9 **(i)** Show that $\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{2}{1\times 3} + \frac{2}{2\times 4} + \ldots + \frac{2}{n(n+2)}.$$ [5]

(iii) Hence find the value of

(a) $\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$, [1]

(b) $\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)}$. [2]

10 The roots of the equation

$$x^3 - 9x^2 + 27x - 29 = 0$$

are denoted by α, β and γ, where α is real and β and γ are complex.

(i) Write down the value of $\alpha + \beta + \gamma$. [1]

(ii) It is given that $\beta = p + iq$, where $q > 0$. Find the value of p, in terms of α. [4]

(iii) Write down the value of $\alpha\beta\gamma$. [1]

(iv) Find the value of q, in terms of α only. [5]

Mark Total

1.	(i) $\frac{2 + 16i - i - 8i^2}{10 + 15i}$ (ii) $\frac{1}{5}(10 + 15i)$ or $2 + 3i$	M1 A1 M1 A1 A1ft	2 3 5	Attempt to multiply correctly Obtain correct answer Multiply numerator & denominator by conjugate Obtain denominator 5 Their part (i) or 10 + 15i derived again / 5
2.	$1^2 = \frac{1}{6} \times 1 \times 2 \times 3$ $\frac{1}{6}n(n + 1)(2n + 1) + (n + 1)^2$ $\frac{1}{6}(n + 1)(n + 2)\{2(n + 1) + 1\}$	B1 M1 DM1 A1 A1	 5 5	Show result true for n = 1or 2 Add next term to given sum formula, any letter OK Attempt to factorise or expand and simplify Correct expression obtained Specific statement of induction conclusion, with no errors seen
3.	(i) $2\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ 2 x 5 – 1 x 2 +3 x -1 5 (ii)	M1 A1 A1 B1ft	 3 1 4	Show correct expansion process, allow sign slips Obtain correct (unsimplified) expression Obtain correct answer State that **M** is non-singular as det **M** non-zero, ft their determinant
4.	$u^2 + 4u + 4$ $u^3 + 6u^2 + 12u + 8$ $u = \sqrt[3]{5}$ $x = 2 + \sqrt[3]{5}$	B1 M1 A1 A1ft A1ft	 5 5	$u + 2$ squared and cubed correctly Substitute these and attempt to simplify Obtain $u^3 - 5 = 0$ or equivalent Correct solution to their equation Obtain 2 + their answer [Decimals score 0/2 of final A marks]

5.	$8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$	M1		Consider the sum of three separate terms
	$8\Sigma r^3 = 2n^2(n+1)^2$	A1		Correct formula stated or used a.e.f.
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated or used a.e.f.
	$2\Sigma r = n(n+1)$	A1		Correct term seen
	$2n^3(n+1)$ **AG**	M1 A1	6 6	Attempt to factorise or expand and simplify Obtain given answer correctly

6.	(i) $\frac{1}{2}\begin{pmatrix} 8 & -2 \\ -3 & 1 \end{pmatrix}$	B1 B1	2	Transpose leading diagonal and negate other diagonal Divide by determinant
	(ii) Either			
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	B1 M1A1		State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ Use this result and obtain $\mathbf{B}^{-1} = \mathbf{C}^{-1}\mathbf{A}$, or equivalent matrix algebra
	Or	M1 A1ft	5	Matrix multn., two elements correct, for any pair All elements correct ft their (i)
	$\frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$	B1		Find $\mathbf{A}^{-1}$
	$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$			
	$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 0 & -2 \\ 5 & 14 \end{pmatrix}$	M1		Premultiply by $\mathbf{A}^{-1}$ stated or implied
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	M1 A1ft		Matrix multn. Two elements correct All elements correct
	Or	A1		Correct $\mathbf{B}^{-1}$
	$\mathbf{AB} = \begin{pmatrix} 2a + c & 2b + d \\ a + 3c & b + 3d \end{pmatrix}$	B1		Find **AB**
	$a = 0, c = 1, b = -0.4, d = 2.8$	M1		Solve one pair of simultaneous equations
		A1A1		Each pair of answers
	$\frac{1}{2}\begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	A1		Correct $\mathbf{B}^{-1}$
			7	

7.	(a) (i) $\sqrt{13}$	B1	1	Obtain correct answer, decimals OK
	(ii)			
		M1		Using $\tan^{-1} b/a$, or equivalent trig allow + or -
	- 0.59	A1		Obtain 0.59
		A1	3	Obtain correct answer
	(b)			
		M1		Express LHS in Cartesian form & equate real and imaginary parts
	$1 - 2i$	A1A1		Obtain $x = 1$ and $y = -2$
		A1	4	Correct answer written as a complex number
	(c)	B1		Sketch of vertical straight line
		B1	2	Through (- 0.5, 0)
			10	
8.	(i)	B1		For correct vertex (2, -2)
	$\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}2\\0\end{pmatrix}$ $\begin{pmatrix}2\\-2\end{pmatrix}$ $\begin{pmatrix}0\\-2\end{pmatrix}$	B1		For all vertices correct
		B1	3	For correct diagram
	(ii) Either $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$	B1,B1		Reflection, in x-axis
		B1		Correct matrix
	$\begin{pmatrix}2&0\\0&2\end{pmatrix}$	B1,B1		Enlargement, centre O s.f.2
		B1	6	Correct matrix
	Or $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$	B1,B1		Reflection, in the y-axis
		B1		Correct matrix
	$\begin{pmatrix}-2&0\\0&-2\end{pmatrix}$	B1,B1		Enlargement, centre O s.f. –2
		B1		Correct matrix
	Or $\begin{pmatrix}2&0\\0&1\end{pmatrix}$	B1,B1		Stretch, in x-direction s.f. 2
		B1		Correct matrix
	$\begin{pmatrix}1&0\\0&-2\end{pmatrix}$	B1,B1		Stretch, in y-direction s.f. -2
		B1		Correct matrix
			9	

9.	(i) $\frac{r+2-r}{r(r+2)}$ $\frac{2}{r(r+2)}$ **AG**	M1 A1	2	Show correct process for subtracting fractions Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Express 1st 3 (or last 3) terms so that cancelling occurs
		A1		Obtain $1+\frac{1}{2}$
		A1		Obtain $-\frac{1}{n+2}, -\frac{1}{n+1}$
	$\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$	A1	5	Obtain correct answer in any form
	(iii) (a) $\frac{3}{2}$	B1ft	1	Obtain value from their sum to n terms
	(b) $\frac{1}{n+1}+\frac{1}{n+2}$	M1 A1 ft	2	Using (iii) (a) – (ii) or method of differences again [$n\to\infty$ is a method error] Obtain answer in any form
			10	
10.	(i) $\alpha+\beta+\gamma=9$	B1	1	
	(ii) $p=\frac{9-\alpha}{2}$	B1 M1 A1 A1	4	State or use other root is p - iq Substitute into (i) Obtain $2p+\alpha=9$ Obtain correct answer a.e.f.
	(iii) $\alpha\beta\gamma=29$	B1	1	
	(iv) $\alpha(p^2+q^2)=29$	M1 A1ft		Substitute into (iii) Obtain unsimplified expression with no i's
		M1		Rearrange to obtain q or q^2
	$q=\sqrt{\frac{29}{\alpha}-\frac{(9-\alpha)^2}{4}}$	M1 A1	5	Substitute their expression for p a.e.f. Obtain correct answer a.e.f.
			11	
	(iv) Alternative method $2p\alpha+p^2+q^2=27$	M1 A1		Substitute into $\alpha\beta+\beta\gamma+\gamma\alpha=27$ Obtain unsimplified expression with no i's
		M1		Rearrange to obtain q or q^2
	$q=\sqrt{27-\frac{(9-\alpha)^2}{4}-\alpha(9-\alpha)}$	M1 A1		Substitute their expression for p a.e.f. Obtain correct answer a.e.f.